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MCE621-Q (MATRIX METHODS IN STRUCTURAL ANALYSIS)
ASSIGNMENT NO.9 BEAMS

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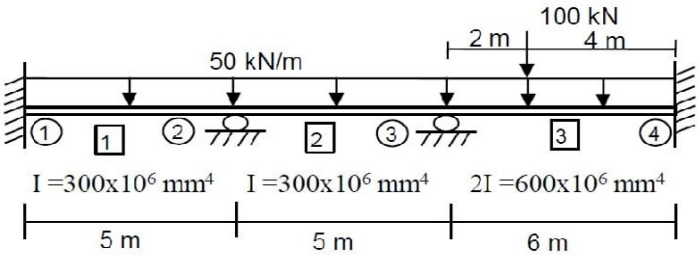
Write your name and signature on top of every page of your solution photo.

1. Determine the reactions and member end forces shown by using the matrix stiffness method.

Ans. d1 = 0.00028 rad, d2 = -0.0011 rad
R1 = 128.98 kN, R2 =234.10 kN, R3 =338.91 kN, R4 = 198.01 kN



E = 200 GPa

Structure Stiffness Matrix

$L_1 = 5$, $E = 200$ GPa, $I_1 = 300\times10^6\ mm^4$ | $L_2 = 5m$, $E = 200$GPa, $I_2 = 300\times10^6\ mm^4$ | $L_3 = 6m$, $E = 200$GPa, $I_3 = 600\times10^6\ mm^4$

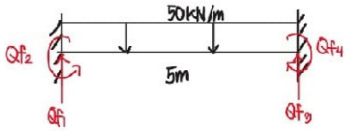
$$k_1 = \frac{200(1000^2)(300\times10^6)\left(\frac{1}{1000^4}\right)}{5^3}\begin{bmatrix} 12 & 6(5) & -12 & 6(5) \\ 6(5) & 4(5)^2 & -6(5) & 2(5)^2 \\ -12(5) & -6(5) & 12 & -6(5) \\ 6(5) & 2(5)^2 & -6(5) & 4(5)^2 \end{bmatrix} = \begin{matrix} 0 & 0 & 0 & 1 \\ \end{matrix}\begin{bmatrix} 5760 & 14400 & -5760 & 14400 \\ 14400 & 48000 & -14400 & 24000 \\ -28800 & -14400 & 5760 & -14400 \\ 14400 & 24000 & -14400 & 48000 \end{bmatrix}\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix}$$

$$k_2 = \frac{200(1000^2)(300\times10^6)\left(\frac{1}{1000^4}\right)}{5^3}\begin{bmatrix} 12 & 6(5) & -12 & 6(5) \\ 6(5) & 4(5)^2 & -6(5) & 2(5)^2 \\ -12(5) & -6(5) & 12 & -6(5) \\ 6(5) & 2(5)^2 & -6(5) & 4(5)^2 \end{bmatrix} = \begin{matrix} 0 & 1 & 0 & 2 \\ \end{matrix}\begin{bmatrix} 5760 & 14400 & -5760 & 14400 \\ 14400 & 48000 & -14400 & 24000 \\ -28800 & -14400 & 5760 & -14400 \\ 14400 & 24000 & -14400 & 48000 \end{bmatrix}\begin{matrix} 0 \\ 1 \\ 0 \\ 2 \end{matrix}$$

$$k_3 = \frac{200(1000^2)(600\times10^6)\left(\frac{1}{1000^4}\right)}{6^3}\begin{bmatrix} 12 & 6(6) & -12 & 6(6) \\ 6(6) & 4(6)^2 & -6(6) & 2(6)^2 \\ -12(6) & -6(6) & 12 & -6(6) \\ 6(6) & 2(6)^2 & -6(6) & 4(6)^2 \end{bmatrix} = \begin{matrix} 0 & 2 & 0 & 0 \\ \end{matrix}\begin{bmatrix} 66666.67 & 20000 & -66666.67 & 20000 \\ 20000 & 80000 & -20000 & 40000 \\ -40000 & -20000 & 66666.67 & -20000 \\ 20000 & 40000 & -20000 & 80000 \end{bmatrix}\begin{matrix} 0 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Fixed-Ended Reactions:

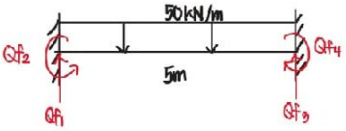
Member 1 +5



$Qf_{11} = \frac{50(5)}{2} = 125kN = Qf_{13}$

$Qf_{12} = \frac{50(5)^2}{12} = 104.167\ kN\cdot m = Qf_{14}$

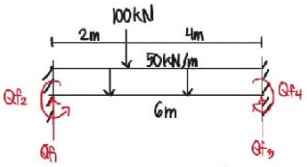
Member 2



$Qf_{21} = \frac{50(5)}{2} = 125kN = Qf_{23}$

$Qf_{22} = \frac{50(5)^2}{12} = 104.167\ kN\cdot m = Qf_{24}$

Member 3



$Qf_{32} = \frac{50(6)^2}{12} + \frac{100(2)(4)^2}{6^2} = 238.889kN\cdot m$

$Qf_{34} = -\left[\frac{50(6)^2}{12} + \frac{100(2)^2(4)}{6^2}\right] = -194.444kN\cdot m$

$\Sigma M_0 = 0$

$Qf_{31}(6) - 238.889 - 100(4) - \frac{50(6)^2}{2} + 194.444 = 0$

$Qf_{31} = 224.074\ kN$

$\Sigma F_y = 0$

$Qf_{33} = 100 + 50(6) - 224.074 = 175.926kN$

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$$P_{f_1} = \begin{bmatrix} 125 \\ 104.167 \\ 125 \\ -104.167 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ (1) \end{matrix} \qquad P_{f_2} = \begin{bmatrix} 125 \\ 104.167 \\ 125 \\ -104.167 \end{bmatrix} \begin{matrix} 0 \\ (1) \\ 0 \\ (2) \end{matrix} \qquad P_{f_3} = \begin{bmatrix} 224.074 \\ 238.889 \\ 175.926 \\ -194.444 \end{bmatrix} \begin{matrix} 0 \\ (2) \\ 0 \\ 0 \end{matrix}$$

Structure Stiffness Σk

1-1	$48000 + 48000 = 96000$
2-1	24000
1-2	24000
2-2	$48000 + 80000 = 128000$

$$S = \begin{matrix} \quad 1 & \quad 2 \\ \end{matrix} \begin{bmatrix} 96000 & 24000 \\ 24000 & 128000 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Fixed-Joint Force Vector

1	$-104.167 + 104.167 = 0$
2	$-104.167 + 238.889 = 134.722$

$$P_f = \begin{bmatrix} 0 \\ 134.722 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint Displacements: $P - P_f = Sd$

$$\begin{bmatrix} 0 \\ 134.722 \end{bmatrix} = \begin{bmatrix} 96000 & 24000 \\ 24000 & 128000 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} d_1 &= 0.000276 \text{ rad} \\ d_2 &= -0.001104 \text{ rad} \end{aligned}} \;//$$

$$d = \begin{bmatrix} 0.000276 \\ -0.001104 \end{bmatrix}$$

$$Q = k \cdot u + Q_f$$

Member 1:

$$F_1 = \begin{bmatrix} 5760 & 14400 & -5760 & 14400 \\ 14400 & 48000 & -14400 & 24000 \\ -28800 & -14400 & 5760 & -14400 \\ 14400 & 24000 & -14400 & 48000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.000276 \end{bmatrix} + \begin{bmatrix} 125 \\ 104.167 \\ 125 \\ -104.167 \end{bmatrix} = \begin{bmatrix} 128.975 \\ 110.792 \\ 121.025 \\ -90.915 \end{bmatrix}$$

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Member 2:

$$F_2 = \begin{bmatrix} 5760 & 14400 & -5760 & 14400 \\ 14400 & 48000 & -14400 & 24000 \\ -28800 & -14400 & 5760 & -14400 \\ 14400 & 24000 & -14400 & 48000 \end{bmatrix} \begin{bmatrix} 0 \\ 0.000276 \\ 0 \\ -0.001104 \end{bmatrix} + \begin{bmatrix} 125 \\ 104.167 \\ 125 \\ -104.167 \end{bmatrix} = \begin{bmatrix} 113.074 \\ 90.915 \\ 136.926 \\ -150.546 \end{bmatrix}$$

Member 3:

$$F_3 = \begin{bmatrix} 66666.67 & 20000 & -66666.67 & 20000 \\ 20000 & 80000 & -20000 & 40000 \\ -40000 & -20000 & 66666.67 & -20000 \\ 20000 & 40000 & -20000 & 80000 \end{bmatrix} \begin{bmatrix} 0 \\ -0.001104 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 224.074 \\ 238.889 \\ 175.926 \\ -194.444 \end{bmatrix} = \begin{bmatrix} 201.988 \\ 150.546 \\ 198.012 \\ -238.616 \end{bmatrix}$$

Member 1 +5 Member 2 Member 3